Metric Based Upscaling for PDEs with a Continuum of Scales

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Abstract

Numerical upscaling of problems with multiple scale structures have attracted increasing attention in recent years. In particular, problems with non-separable scales pose a great challenge to mathematical analysis and simulation. Most existing methods are either based on the assumption of scale separation or heuristic arguments.

In this talk, we present rigorous results on homogenization of partial differential equations with L^∞ coefficients which allow for a continuum of spatial and temporal scales. We propose a new type of compensation phenomena for elliptic, parabolic, and hyperbolic equations. The main idea is: through the use of the so-called "harmonic coordinates" ("caloric coordinates" in the parabolic case), the solutions of the differential equations have one more degree of differentiability. It has been deduced from this compensation phenomenon that numerical homogenization methods formulated as oscillating finite elements can converge in the presence of a continuum of scales, if one uses global caloric coordinates to obtain the test functions instead of using solutions of a local cell problem.