Abstract

Let (S_n) be a sequence of real or complex numbers converging to a limit S. If the convergence is slow, and if one has no access to the process producing the sequence (that is, if it is a black box), (S_n) can be transformed into a new sequence (T_n) converging to the same limit by a *sequence transformation* T. Under some assumptions on (S_n) and T, (T_n) can converge to S faster than (S_n), that is

$$\lim_{n\to\infty} \frac{T_n - S}{S_n - S} = 0.$$

The idea behind a sequence transformation is *extrapolation to the limit*. It is assumed that (S_n) behaves as a model sequence $(\widetilde{S}_n)$ depending on p parameters, and belonging to a given class $\mathcal{K}_\mathcal{T}$ of sequences. These p parameters are obtained by interpolation, requiring that $S_i = \widetilde{S}_i$ for $i = n, \ldots, n+p-1$, thus defining a unique model sequence in $\mathcal{K}_\mathcal{T}$ depending on the index n (the first index used in the interpolation process). Then, the limit of this model sequence is considered as an approximation of S. Since this limit depends on n, it is denoted by T_n, and, therefore, the sequence (S_n) has been transformed into the new sequence (T_n).

The most important sequence transformations will be reviewed and their properties explained.