数学与系统科学研究院

计算数学所学术报告

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报告题目：

Plane Wave Discontinuous Galerkin Methods

邀请人： 陈志明 研究员

报告时间：**2015** 年 **6** 月 **5** 日（周五）

上午 **10:00~11:00**

报告地点： 数学院南楼七层

702 会议室

Abstract:

This lecture reviews the current convergence theory for a special class of Trefftz-type discontinuous Galerkin (TDG) methods that rely on plane waves for approximating solutions of the homogeneous Helmholtz equation $-\Delta u-\omega^2 u = 0$ locally. These methods have been designed as a cure for the notorious pollution e_ect that haunts standard low-order Galerkin schemes for the simulation of wave propagation. A prominent representative is the so-called ultra-weak variational (UWVF) invented by Cessenat and Despres.

A powerful tool in the analysis of plane wave discontinuous Galerkin methods (PWDG) are duality techniques borrowed from least squares methods. They paved the way for quasioptimality results. Together with new approximation estimates for plane waves [3, 4], this allowed detailed a priori predictions of convergence. However, the early versions of the theory [1] were confined to convex domains, and, even worse, could not accommodated locally refined meshes, which is very unfortunate, because numerical experience suggests that PWDG should be used on such meshes. In short, the sophisticated hp-refinement strategy that ensures exponential convergence (in the number of degrees of freedom) for classical polynomial Galerkin finite element approximation of second-order elliptic boundary value problem should also be adopted for PWDG.

Only recently, the theory could be extended to the full hp-setting that involves families of trial spaces generated by dyadic mesh re_nement towards corners combined with a global increase of the number of plane waves used in each element. The key idea is to make clever use of the freedom to choose ux parameters in the DG method, which makes it possible to establish quasi-optimality without assuming quasi-uniformity. The second ingredient is new estimates for the approximation of analytic Helmholtz solutions by plane waves. Thus, exponential convergence of hp-PWDG for the Helmholtz equation in 2D on domains with piecewise analytic boundaries could be established [2].

References

[1] R. Hiptmair, A. Moiola, and I. Perugia, Plane wave discontinuous Galerkin methods for the 2d Helmholtz equation: Analysis of the p-version, SIAM J. Numer. Anal., 49 (2011), pp. 264{284.

[2] , Plane wave discontinuous Galerkin methods: Exponential convergence of the hp-version, Report 2013-31, SAM, ETH Z urich, Switzerland, 2013. To appear in Found.

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[3] A. Moiola, R. Hiptmair, and I. Perugia, Plane wave approximation of homogeneous Helmholtz solutions, ZAMP, 62 (2011), pp. 809{837.

[4] , Vekua theory for the Helmholtz operator, ZAMP, 62 (2011), pp. 779{807.

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