

数学与系统科学研究院  
计算数学所系列学术报告

报告人: **Prof. Zhijun Qiao**

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报告题目:

**Integrable Peakon Systems**

邀请人: 胡星标 研究员  
常向科 博士

报告时间及地点:

2017 年 7 月 17 日 (周一) 下午 15:00-17:00 数学院思源楼 705 教室

2017 年 7 月 18 日 (周二) 上午 9:30-11:30 数学院思源楼 705 教室

2017 年 7 月 18 日 (周二) 下午 15:00-17:00 数学院思源楼 705 教室

2017 年 7 月 19 日 (周三) 下午 15:00-17:00 数学院思源楼 705 教室

2017 年 7 月 20 日 (周四) 下午 15:00-17:00 数学院思源楼 705 教室

2017 年 7 月 21 日 (周五) 下午 15:00-17:00 数学院思源楼 705 教室

## **Abstract:**

### **1. Introduction to Peakons and CH equation**

**A short review will be given to introduce how we have a peakon solution for an integrable system. We will start from the standard CH equation to explain the peakon solution actually comes from a special type formulation. Then we will extend the results to the dispersion type CH equation and the whole b-family equation. Similar strategy is applied to the integrable cubic CH equation (MCH/FORQ equation), but foundation is completely dependent on the product of delta function and sign-squared function. We will show some very interesting results which may lead to explore the integrability study for multi-peakon system.**

### **2. DP equation and 3 by 3 Lax pair**

**In this lecture, we will discuss integrability of multi-peakon systems generated from the quadratic b-family equation. The CH case will be first addressed, but integrability depends on an artificial assumption. Some open problems pop up for the DP equation and the b-family system as well.**

### **3. Quadratic peakon system and multi-peakon solutions**

**In the b-family system, only the CH (b=2) and the DP (b=3) equations is found integrable so far. In this lecture, we will display some significant difference between the CH and DP. The DP equation can be extended to the whole integrable hierarchy through inverse of recursion operator. Some exact solutions are addressed for the positive and negative order members in the hierarchy.**

#### **4. Cubic peakon systems**

**In this lecture, two typical cubic integrable peakon systems are explained. One is MCH/FORQ and the other NE. Lax pair and their multi-peakon systems are discussed. But integrability of N-peakon-dynamical systems are still under investigation under the canonical Hamiltonian structure in the classical sense.**

#### **5. Two component peakon systems**

**In this lecture, we will majorly focus on cubic coupled systems though a general N-component peakon system is proposed. All the systems are shown integrable in the sense of Lax pair. References for this lecture involved include Xia and Q, Journal of Geometry and Physics, 107 (2016), 35-44, Proc. R. Soc. A, 471 (2015), 20140750, and Xia, Q.**

and Zhou, *Studies in Applied Mathematics*, 135 (2015),  
248-276

## 6. Negative flows and Open problems

So far, almost all integrable peakon equations come from the negative flows of integrable hierarchy. One of important issue is inverse of recursion operator. I will show some earlier work on integrable negative flows in earlier 1990's. We will see some important equations (including CH, DP, short pulse equations, etc) would be produced through the negative order hierarchy. Lastly, some open problems will be addressed.

**欢迎大家参加！**