数学与系统科学研究院

计算数学所学术报告

报告人： **张庆海教授**

(浙江大学数学与统计学院)

系列报告：第一讲

报告题目：

Introduction to interface tracking methods

邀请人： 郑伟英研究员

报告时间：**2018 年 3 月 17 日（周六）**

下午 14:30--17:30

报告地点： 数学院科技综合楼

Z311 报告厅

报告摘要：附后

欢迎大家参加！

Lectures on Interface Tracking: Theory, Algorithms, and Software

Qinghai Zhang (张庆海)
School of Mathematical Sciences
Zhejiang University

2018 MAR 17,18,19,24,25,26 @ Chinese Academy of Sciences, AMSS

In numerically studying multiphase flows with moving boundaries, interface tracking (IT) is a fundamental problem, as the accuracy of simulated physical variables depends largely on the interface location and the derived quantities such as interface normal and curvature.

Most current IT methods are at best second-order accurate. Among other thing, the most serious disadvantage is probably the absence of a systematic framework to analyze their convergence rates and to answer open questions on accuracy deteriorations that cannot be explained by the current approaches. Based on the donating region theory [2, 1, 3, 6], we resolve this deficit by proposing a generic framework called MARS [8], Mapping and Adjusting Regular semialgebraic sets.

Using MARS, we formally proved the second-order accuracy of Volume-of-Fluid (VOF) methods, clarified many subtle issues such as the accuracy deterioration caused by local C^1 discontinuities, and analyzed other explicit IT methods such as moment-of-fluid methods and front-tracking methods. Inspired by MARS, we also proposed a new IT method called the iPAM method [7, 5], which achieved 4th-, 6th-, and 8th-order accuracy for an arbitrary number of phases. For the classic vortex-shear tests, solutions of iPAM are close to machine precision on a 128-by-128 grid; it could also be vastly superior to VOF methods in terms of efficiency.

MARS appears to be the first systematic framework for analyzing explicit IT methods, and the iPAM method the first IT method with fourth and higher-order accuracy. Other advantages of MARS and iPAM include (i) the independence of the high-order accuracy of dynamic C^1 discontinuities, (ii) rigorous volume conservation, (iii) high efficiency, (iv) direct applicability to both structured and unstructured grids, (v) adaptive resolution of the interface, (vi) versatility of decoupling the resolution of interface from that of the bulk flow, and (vii) 4th- and higher-order accuracy of curvature estimation [8, 4].

This lecture series aims to tell a concise and coherent story out of the lengthy and meandering development of MARS. We will review current IT methods, touch upon some prerequisites, elaborate on several important theorems of MARS, explain details of iPAM algorithms, and discuss designs of the companion matlab packages.

Tentatively, the contents to be covered are as follows.

1. Introduction ($\sim$ 1 hour)
   the Level set method and the front tracking method; the VOF method and its variants; pros and cons of current methods; why bother with fourth- or higher-order methods? flux approximation and the new of a generic analytic framework.

2. Donating regions: theory and algorithms ($\sim$ 5 hours)
   flow maps, streaklines, the winding number, Hopf theorem, and Reynolds transport theorem; the index-by-index equivalence of flux sets and donating regions; the flux identity for scalar hyperbolic conservation law; highly accurately Lagrangian flux calculation via Gauss integration on splinegons; application to the analysis of VOF methods.

3. MARS ($\sim$ 3 hours)
   the topological space of regular open sets; semianalytic sets; modeling physically meaningful material regions with the Yin space; formalizing the IT problem in the Yin space; the definition of an IT method and its accuracy; analysis of the IT errors.

4. iPAM ($\sim$ 6 hours)
   the PAM method; exact volume conservation with polygon ear removal; the iPAM method; splinegon clipping; Boolean algebra on the Yin space via posets of oriented Jordan curves; the matlab package implementing iPAM.

5. HFES for curvature estimation ($\sim$ 3 hours)
   the height function of a planar curve; signed curvature and the local direction of convexity; representing the local information via volume fractions; the sweeping line; a unified formula for curvature estimation; the existence of the best estimation and how to attain it without painful refinements; the analysis showing advantages of high-order methods; the matlab package implementing HFES.

As for the prerequisites, it should suffice to read preliminaries sections in the reprints on the following webpage.
`www.researchgate.net/profile/Qinghai_Zhang2`

References

[1] Q. Zhang. On a family of unsplit advection algorithms for volume-of-fluid methods. *SIAM J. Numer. Anal.*, 51(5):2822–50, 2013.

[2] Q. Zhang. On donating regions: Lagrangian flux through a fixed curve. *SIAM Review*, 55(3):443–61, 2013.

[3] Q. Zhang. On generalized donating regions: Classifying lagrangian fluxing particles through a fixed curve in the plane. *J. Math. Anal. Appl.*, 424(2):861–877, 2015.

[4] Q. Zhang. HFES: a height function method with explicit input and signed output for high-order estimations of curvature and unit vectors of planar curves. *SIAM J. Numer. Anal.*, 55:1024–1056, 2017.

[5] Q. Zhang. A cubic MARS method for fourth-, sixth-, and eighth-order interface tracking. *submitted for publication*, 2018.

[6] Q. Zhang and L. Ding. A flux identity of scalar conservation laws for Lagrangian flux calculation via donating regions. *submitted for publication*, 2018.

[7] Q. Zhang and A. Fogelson. Fourth-order interface tracking in two dimensions via an improved polygonal area mapping method. *SIAM J. Sci. Comput.*, 36:A2369–A2400, 2014.

[8] Q. Zhang and A. Fogelson. MARS: An analytic framework of interface tracking via mapping and adjusting regular semi-algebraic sets. *SIAM J. Numer. Anal.*, 54:530–560, 2016.