# Numerical Analysis of an Integral Equation for Macroscopic Simulations of Micromagnetics for the Large-Soft Limit

Carsten Carstensen
Department of Mathematics Humboldt-Universität zu Berlin, D-10099 Berlin
cc@math.hu-berlin.de

**Abstract**

The large body limit in the Landau-Lifshitz equations of micromagnetics [1] yields a macroscopic model without exchange energy and with convexified side conditions for the macroscopic magnetisation vectors. Its Euler Lagrange equations $(P)$ read: Given a magnetic body $\Omega \subseteq \mathcal{R}^d$, $d = 2, 3$, an exterior field $\mathbf{f} \in L2(\Omega)^d$, and the convexified anisotropy density $\phi^{**} : \mathcal{R}^d \to \mathcal{R}_{\geq 0}$, find a magnetization $\mathbf{m} \in L2(\Omega)^d$ and a Lagrange multiplier $\lambda \in L2(\Omega)$ such that a.e. in $\Omega$

$$\begin{aligned} &\nabla u + D\phi^{**}(\mathbf{m}) + \lambda \mathbf{m} = \mathbf{f}, \\ &|\mathbf{m}| \leq 1, \quad \lambda \geq 0, \\ &\lambda(1 - |\mathbf{m}|) = 0. \end{aligned} \tag{1}$$

The potential $u \in H^1_{\ell oc}(\mathcal{R}^d)$ solves the (Maxwell) equations

$$\nabla u \in L^2(\mathcal{R}^d)^d \text{ and } \operatorname{div}(-\nabla u + \mathbf{m}) = 0 \text{ in } \mathcal{D}'(\mathcal{R}^d) \tag{2}$$

in the entire space. It therefore appears natural to recast the associated far field energy into an integral operator $\mathcal{L}$ which maps $\mathbf{m}$ to the corresponding potential $u$.

The proposed numerical scheme involves the operator $\mathcal{L}$ and replaces pointwise side-condition $|\mathbf{m}| \leq 1$ by a penalization strategy. Given a triangulation $\mathcal{T}$, the induced space of piecewise constant functions $P_0(\mathcal{T})$ on $\Omega$, and a penalization parameter $\varepsilon > 0$, the discrete penalized problem $(P_{\varepsilon,h})$ reads: Find $\mathbf{m}_h \in P_0(\mathcal{T})^d$ such that for all $\nu_h \in P_0(\mathcal{T})^d$

$$\langle \nabla(\mathcal{L}\mathbf{m}_h) + D\phi^{**}(\mathbf{m}_h) + \lambda_h \mathbf{m}_h \,; \nu_h \rangle_{L2(\Omega)} = \langle \mathbf{f} \,; \nu_h \rangle_{L2(\Omega)} \tag{3}$$

with

$$\lambda_h := \varepsilon^{-1} \, \frac{\max\{0, |\mathbf{m}_h| - 1\}}{|\mathbf{m}_h|} \in P_0(\mathcal{T}).$$

Numerical aspects addressed in the presentation include the integration of the matrices with quadrature rules and hierarchical matrices as well as a priori and a posteriori error control with a reliability-efficiency gap and adaptive mesh-design. Surprisingly, a comparison with finite element approximations [2,3] indicates that local mesh-adaptation is not really required for the new model $(P_{\varepsilon,h})$ in many numerical examples.

The presentation is joint work with Dr. Dirk Praetorius and supported by the FWF.

**Reference**:

[1] A. DeSimone: *Energy Minimizers for Large Ferromagnetic Bodies*, Arch. Rational Mech. Anal. 125 (1993), 99-143.

[2] C. Carstensen, A. Prohl: *Numerical Analysis of Relaxed Micrmagnetics by Penalised Finite Elements*, Numer. Math. 90 (2001), 65-99.

[3] D. Praetorius: *Analysis, Numerik und Simulation eines relaxierten Modellproblems zum Mikromagnetismus*, Doctorial thesis, Vienna University of Technology 2003.

[4] C. Carstensen, D. Praetorius: *Effective stimulation of a macroscopic model for stationary micromagnetics*, Preprint NI03070-CPD, Isaac Newton Institute for Mathematical Sciences (available at `http://www.newton.cam.ac.uk/preprints/NI03070.pdf`), submitted 2003.

[5] C. Carstensen, D. Praetorius: *Numerical Analysis for a Macroscopic Model in Micromagnetics*, in preparation 2004.