

ADAPTIVE SPARSE TENSOR PRODUCT METHODS FOR RADIATIVE TRANSFER

R. HIPTMAIR¹, C. SCHWAB¹, AND G. WIDMER¹

(1) Seminar for Applied Mathematics, ETH Zürich, e-mail: {lastname}@sam.math.ethz.ch

The linear radiative transfer equation, a partial differential equation for the radiation intensity $u(\mathbf{x}, \mathbf{s})$, with independent variables $\mathbf{x} \in D \subset \mathbb{R}^n$ in the physical domain D of dimension $n = 2, 3$, and angular variable $\mathbf{s} \in S^2 := \{\mathbf{y} \in \mathbb{R}^3 : |\mathbf{y}| = 1\}$, is solved in the $n+2$ -dimensional computational domain $D \times S^2$.

We propose an adaptive multilevel Galerkin FEM for its numerical solution. Our approach is based on a) a stabilized variational formulation of the transport operator, b) on so-called sparse tensor products of two hierarchic families of Finite Element spaces in $H^1(D)$ and in $L^2(S^2)$, respectively and c) on wavelet thresholding techniques to adapt the discretization to the underlying problem.

An a-priori error analysis shows, under strong regularity assumptions on the solution, that the sparse tensor product method is clearly superior to a discrete ordinates method, as it converges with essentially optimal asymptotic rates while its complexity grows essentially only as that for a linear transport problem in \mathbb{R}^n . Numerical experiments for $n = 2$ on a set of example problems agree with the convergence and complexity analysis of the method and show that introducing adaptivity can improve performance in terms of accuracy vs. number of degrees even further.

References

- [1] G. WIDMER, R. HIPTMAIR, AND C. SCHWAB, *Sparse adaptive finite elements for radiative transfer*, Tech. Rep. 2007-01, SAM, ETH Zürich, Zürich, Switzerland, January 2007, submitted to JCP.