Convergence of Shortley-Weller Approximations for a Class of Second-Order Elliptic Equations with Singular Solutions

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Abstract

We consider in this talk the Dirichlet problem of a class of 2nd order elliptic equations:

$$\begin{cases} \mathcal{L}u \equiv -a(x,y)\dfrac{\partial^2 u}{\partial x^2} - 2b(x,y)\dfrac{\partial^2 u}{\partial x \partial y} - c(x,y)\dfrac{\partial^2 u}{\partial y^2} - \mathbf{d}(x,y)\cdot\nabla u - e(x,y)u = f(x,y) \quad \text{in} \quad \Omega, \\ u = g(x,y) \quad \text{on} \quad \Gamma = \partial\Omega. \end{cases}$$

where functions $a, b, c > 0$, $ac - b^2 > 0$ and $\mathbf{d} = (d_1(x,y), d_2(x,y))$ is bounded on $\bar{\Omega}$.

Assume that the above problem has a unique solution $u(x,y) \in C^4$ and that u has specified singular derivatives near the boundary Γ, we propose the Shortley-Weller difference method to the approximation of the problem and obtain the convergence conclusion for the approximate solution U:

(i) If $r = \sigma(p+1) \leq \sigma(p_{max}+1) < 2$, then $|u(P) - U(P)| \leq O(h^r)$, $\forall P \in \Omega_h$.

(ii) If $\sigma(p_{max}+1) \geq 2$, and $r = \sigma(p+1) = 2$, then $|u(P) - U(P)| \leq O(h^2 \log \frac{1}{h})$, $\forall P \in \Omega_h$.

where p_{max} is determined by the evaluation of a, b and c on Ω_h. Finally, we use numerical examples to confirm our theory. This is a joint work with Yong-Qing Wu