

数学与系统科学研究院

计算数学所学术报告

报告人: **Dr. Tino Ullrich**

(*Hausdorff Center for Mathematics, University of Bonn*)

报告题目:

The Gelfand widths of l_p -balls for

$0 < p \leq 1$ **Lower bounds for sparse recovery**

邀请人: 许志强副研究员

报告时间: **2011年2月25日(周五)**

上午 10:30

报告地点: **科技综合楼三层 301**

计算数学所小报告厅

欢迎大家参加!

The Gelfand widths of ℓ_p -balls for $0 < p \leq 1$ Lower bounds for sparse recovery

TINO ULLRICH

tino.ullrich@hcm.uni-bonn.de

Hausdorff Center for Mathematics, University of Bonn

Joint work with:

Simon Foucart (Paris), Alain Pajor (Marne-la-Vallée), Holger Rauhut (Bonn)

For a subset K of \mathbb{R}^N (quasi-)normed by $\|\cdot\|_X$, the Gelfand widths are defined as the numbers

$$d^m(K, X) := \inf_{A \in \mathbb{R}^{m \times N}} \sup_{v \in K \cap \ker A} \|v\|_X, \quad m < N,$$

and represent an important concept in classical and modern approximation and complexity theory. They have found recent interest in the rapidly emerging field of compressive sensing because they give general performance bounds for sparse recovery methods. Since vectors in ℓ_p -balls $K = B_p^N$, $0 < p \leq 1$, can be well-approximated by sparse vectors in $X = \ell_q^N$ if $0 < p < q$, the Gelfand widths of such balls are particularly relevant in this context. In substantial papers from the 1970s and 80s due to Kashin, Gluskin, and Garnaev, upper and lower estimates for the Gelfand widths of ℓ_1 -balls are provided. Donoho extends these estimates to the Gelfand widths of ℓ_p -balls with $p < 1$. For $0 < p \leq 1$ and $p < q \leq 2$ it holds

$$\min \left\{ 1, \frac{\ln(N/m) + 1}{m} \right\}^{1/p-1/q} \lesssim d^m(B_p^N, \ell_q^N) \lesssim \min \left\{ 1, \frac{\ln(N/m) + 1}{m} \right\}^{1/p-1/q}. \quad (1)$$

Unfortunately, his proof of the lower bound contains a gap and so the result remained unproved. In this talk we show how to use compressive sensing methods to establish this lower bound in a more intuitive way. Our method is new even for the case $p = 1$. In the remaining time we comment on the proof of the upper bound based on RIP estimates. Our techniques also provide the same sharp asymptotic behavior for the Gelfand widths of weak- ℓ_p -balls.