

数学与系统科学研究院

计算数学所学术报告

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报告题目:

**Making Computational Integration  
Accessible**

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**计算数学所报告厅**

# Abstract:

The constructive definition usually begins with a function  $f$ , then by the process of using Riemann sums and limits, we arrive the definition of the integral of  $f$ . On the other hand, a descriptive definition starts with a primitive  $F$  satisfying certain condition(s) such as  $F' = f$  and  $F$  is absolutely continuous if  $f$  is Lebesgue integrable, and  $F$  is generalized absolutely continuous if  $f$  is Henstock-Kurzweil integrable. For descriptive integrals, the deficiency is that we need to know primitive  $F$  for which  $F' = f$  and satisfying some properties. For constructive integration, we proposed using an uneven partition to get a broader family functions which includes some improper Riemann integrals. In this paper, we describe how we can make use of the Fundamental Theorem of Calculus and the constructive definition to reach a description definition for some improper Riemann integrable functions that are monotonic or highly oscillating with singularity on one end. Under the Henstock-Kurzweil integral, the Fundamental Theorem of Calculus holds in one dimension and Divergence Theorem holds in higher dimensions. In particular, we explore the followings:

1. If the primitive function (anti-derivative) for a given highly oscillating function  $f$  is not known, we will see how we can use countably many extensions of a Riemann integral to recover its primitive function. We can do this in higher dimension too. However, we need to take Fubini's Theorem into consideration..
2. Fundamental Theorem of Calculus in 1D or Divergence Theorem in higher dimension are bonuses but not the ultimate goal of applying effective way of computing an integral. There are cases where there is no  $F$  such that  $F' = f$  or no vector field  $F$  such that  $\text{div } F = f$ , then using computation quadratures in 1D and 2D respectively mentioned here are important to recover  $F$ .
3. We use computational method in 2D and check if Fubini's Theorem is met before we apply iterated integration.

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