数学与系统科学研究院

计算数学所学术报告

报告人： **Dr. Kaibo Hu**

（ ***Peking University*** ）

报告题目：

Nodal Finite Element de Rham Complexes

邀请人： 张硕 博士

报告时间：2017 年 6 月 22 日（周四）

上午 **10:00-11:00**

报告地点： 科技综合楼三层

305 会议室

Abstract： 附后

欢迎大家参加！

Nodal Finite Element de Rham Complexes

Kaibo Hu
Peking University

The de Rham complex

$$\mathbb{R} \longrightarrow H(\mathrm{grad}, \Omega) \xrightarrow{\mathrm{grad}} H(\mathrm{curl}, \Omega) \xrightarrow{\mathrm{curl}} H(\mathrm{div}, \Omega) \xrightarrow{\mathrm{div}} L^2(\Omega) \longrightarrow 0$$

describes compatibility conditions and potential functions in electromagnetism and diffusion problems. Finite element discretizations of the de Rham complex are well studied, and the classical Lagrange, Nédélec (1st, 2nd), Raviart-Thomas, Brezzi-Douglas-Marini elements can be summarized in a "finite element periodic table" [Arnold & Logg, 2014] in any spacial dimension with any polynomial degree.

In this talk, we add a new regularity parameter $r = 0, 1, 2$ to extend the periodic table. The new complexes are exact in contractible domains. Some of the elements have Lagrange or Hermite type nodal basis, which is more canonical and easy to implement for high order methods. The number of global degrees of freedom is reduced compared with classical de Rham elements. Combining with Bernstein-Gelfand-Gelfand (BGG) type constructions, we match finite element de Rham complexes with different continuities to construct tensor finite elements with strongly imposed constraints.

This is a joint work with Snorre H. Christiansen and Jun Hu.

	$k=0$	$k=1$	$k=2$
$r=0$	Lagrange $\mathcal{P}_p\Lambda^0(\mathcal{T}_h^2)$	BDM $\mathcal{P}_{p-1}\Lambda^1(\mathcal{T}_h^2)$	DG $\mathcal{P}_{p-2}\Lambda^2(\mathcal{T}_h^2)$
$r=1$	Hermite $\mathcal{P}_{1,p}\Lambda^0(\mathcal{T}_h^2)$	Stenberg $\mathcal{P}_{1,p-1}\Lambda^1(\mathcal{T}_h^2)$	DG $\mathcal{P}_{1,p-2}\Lambda^2(\mathcal{T}_h^2)$
$r=2$	Argyris $\mathcal{P}_{2,p}\Lambda^0(\mathcal{T}_h^2)$	vector Hermite $\mathcal{P}_{2,p-1}\Lambda^1(\mathcal{T}_h^2)$	Falk-Neilan $\mathcal{P}_{2,p-2}\Lambda^2(\mathcal{T}_h^2)$

Table 1: 2D families.

	$k=0$	$k=1$	$k=2$	$k=3$
$r=0$	Lagrange $\mathcal{P}_p\Lambda^0(\mathcal{T}_h^3)$	Nédélec $\mathcal{P}_{p-1}\Lambda^1(\mathcal{T}_h^3)$	BDM $\mathcal{P}_{p-2}\Lambda^2(\mathcal{T}_h^3)$	DG $\mathcal{P}_{p-3}\Lambda^3(\mathcal{T}_h^3)$
$r=1$	Hermite $\mathcal{P}_{1,p}\Lambda^0(\mathcal{T}_h^3)$	new, $\mathcal{P}_{1,p-1}\Lambda^1(\mathcal{T}_h^3)$	BDM $\mathcal{P}_{1,p-2}\Lambda^2(\mathcal{T}_h^3)$	DG $\mathcal{P}_{1,p-3}\Lambda^3(\mathcal{T}_h^3)$
$r=2$	(scalar) 3D Neilan velocity $\mathcal{P}_{2,p}\Lambda^0(\mathcal{T}_h^3)$	new, $\mathcal{P}_{2,p-1}\Lambda^1(\mathcal{T}_h^3)$	Stenberg $\mathcal{P}_{2,p-2}\Lambda^2(\mathcal{T}_h^3)$	DG $\mathcal{P}_{2,p-3}\Lambda^3(\mathcal{T}_h^3)$

Table 2: 3D families.