数学与系统科学研究院

计算数学所学术报告

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（ 中科院软件所 ）

报告题目：

Novel spectral and spectral element methods for a class of singular eigenvalue problems

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报告时间：**2017** 年 **7** 月 **14** 日（周五）

下午 **16:00-17:00**

报告地点：　数学院南楼九层

902 教室

Abstract:

In this talk, we first consider the eigenvalue problem of the Schr\"{o}dinger equation,

\begin{align*} -\Delta u+ \frac{c_3^2}{|\bx|^3} u + \frac{c_2^2}{|\bx|^2}u + \frac{c_1^2}{|\bx|} u = \lambda u \qquad \text{in } \Omega \subset \mathbb{R}^d, \end{align*}

subject to appropriate boundary conditions, where ${\bs 0} \in \overline{\Omega}$.

We start from the situations for Ω being the unit ball or the whole space of any dimension, in which the exact solution in the radial direction can be expressed by Bessel functions of fractional degrees. This knowledge helps us to design some novel spectral methods in both cases by modifying the polynomial basis to fit the singularities of the eigenfunctions.

Then we move to circular sectors in the two-dimensional setting. Again the radial direction can be expressed by Bessel functions of fractional degrees. Only in the tangential direction some modifications are needed from stage one. Moreover, we extend the idea to arbitrary polygonal domains.

We propose a mortar spectral element approach: a polygonal domain is decomposed into several subdomains with each singular corner including the origin covered by a circular sector, in which origin and corner singularities are handled similarly as in the former stages, and the remaining domains are either a standard quadrilateral/triangle or a quadrilateral/triangle with a circular edge, in which the traditional polynomial based spectral method is applied. All subdomains are linked by mortar elements.

Finally, we extend our novel spectral methods for solving the following eigenvalue problem for the mathematical model of fuel cells,

\begin{align*} -\frac{\partial^2 u}{\partial x_1^2} - \frac{1}{x_1^2}\frac{\partial^2 u}{\partial x_2^2}= \lambda u \qquad \text{in } (0,1)^2. \end{align*}

In all situations, exponential convergence rates are achieved. Numerical experiments indicate that our new methods are superior to standard polynomial based spectral (or spectral element) methods and hp-adaptive methods. Our study offers a new and effective way to handle eigenvalue problems of some singular elliptic operators including the Laplacian operator on polygonal domains with reentrant corners.

欢迎大家参加！